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Creating Atom-Molecule Entanglement from a Two-Species Atomic Bose Condensate

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Abstract In this paper, we propose a feasible scheme to create macroscopic atom-molecule entanglement from a two-species atomic Bose-Einstein condensate, focusing on the role of the initial populations imbalance Δ of the two species. We find that, by tuning the value of Δ and/or the initial quantum statistics of the atoms, the atom-heteronuclear molecule entanglement can be indeed realized.

Keywords Bose-Einstein condensate · Quantum entanglement · Matter wave

Since the realization of Bose-Einstein condensates (BEC) in dilute atomic gases [1–3], there are many interests in the generation of coherent matter waves [4–9] and their various novel applications [10, 11]. One of such ongoing efforts is the creation of an atom laser for the purpose of high-precision matter-wave interferometry. Due to their much smaller wavelength, the use of the atom laser can lead to a substantial increase of interferometer sensitivities as compared to their optical counterparts. Recently, Haine and Hope proposed a method to create even a squeezed atom laser by using a squeezed light to out-couple atoms from a trapped condensate, and thus the quantum noise in one field quadrature can be further reduced at the cost of increased noise in another quadrature [12]. Subsequently, Haine and his co-workers proposed another feasible scheme to produce the controllable atom-light entanglement still by using a squeezed input light [13]. The basic mechanism of these works is the quantum transfer from the input nonclassical light to the output atoms or atom-photon pairs. In fact, the creation of an entangled atom-photon or atom-atom pairs and their potential applications in current quantum information science have been studied extensively [14–16], such as the works based on molecular down-conversion [17], spin-exchange collisions [18, 19], or the versatile technique of quantum transfer [13]. Therefore a natural question arise: can these

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technique be generalized to create the entanglement between the atom-molecule pairs? In view of the rapid advances of the creation and manipulations of ultracold molecules in current experiments, a non-classical atom-molecule laser will be interesting for, e.g., a hybrid matter-wave interferometer.

Recently Borca and his co-workers proposed to create an atom-homonuclear molecule laser by using the stimulated three-body recombination [20, 21]. The possibility to create the quantum entanglement or macroscopic superposition of atoms and molecules was also studied by using the bosonic [20–23] or fermionic atoms [24]. However, these works have been focus on the atom-homonuclear molecule pairs. We note that in recent years the formation of the heteronuclear dimers was also observed at ultracold energies by using the two-species atomic sample via magnetic Feshbach resonance (FR) or optical photo-association (PA) methods [25–28]. In view of their novel properties like the large electric dipole moments, the heteronuclear molecular sample is expected to lead to some new phenomena [26, 29, 30] with potential applications in, e.g., quantum computing [31, 32] and ultrahigh precision measurement [33, 34].

In this paper, we propose a feasible scheme to create quantum entanglement between the ultracold atoms and the heteronuclear dimers by using a Raman-type atom laser coupler coupled with the FR-induced atom-dimer conversion. Our system is starting from a two-species atomic Bose condensate, which has been extensively studied in recent literatures, such as the dynamical creation of fractionalized vortices and vortex lattices [35, 36], the exact soliton-on-plane-wave solutions [37], or the phase diagram of two-species Bose condensates in an optical lattice [38]. The FR occurs when the energy of the atomic scattering state is tuned into degeneracy with that of a bound molecular state. Our scheme here is actually a generalization of previously proposed scheme to create macroscopic atom-molecule entanglement based on homonuclear dimers formation [22–24, 39]. The new feature here, however, is the role of initial populations imbalance of the two-species atoms on the mutual coherence of the closed-channel atoms and heteronuclear molecules. One of our main results is that, for different cases of initial populations imbalance $\Delta \equiv N_A(0) - N_B(0)$, where $N_A(0)$ is the initial populations of atoms A and $N_B(0)$ for atoms B, the atom-molecule entanglement can be realized due to the inherent nonlinearity (for small Δ) or the quantum transfer process (for large negative Δ).

The basic model of our scheme is illustrated in Fig. 1. Specifically, we consider the coherent creation of heteronuclear molecules via the magnetic FR in a two-species atomic condensate, which is accompanied by the two-photon atomic transition $|1\rangle \rightarrow |2\rangle$ (with an

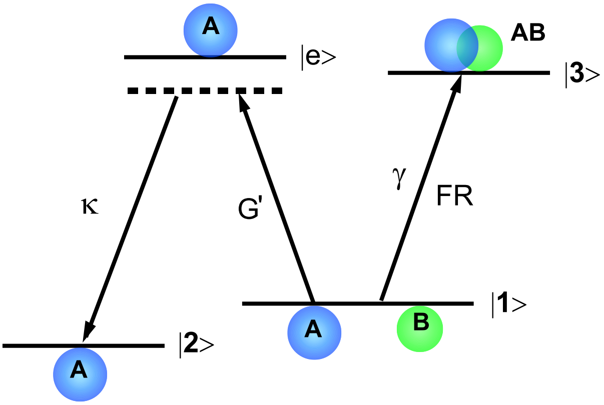

Fig. 1 (Color online) Schematic illustration of our scheme to create quantum entanglement of atoms A and heteronuclear molecules AB from a two-species atomic condensate (see the text for a detailed description of the mechanism)

intermediate state $|e\rangle$). The heteronuclear molecules AB in state $|3\rangle$ are created by applying the suitable magnetic FR sweeping pulses with the magnetic coupling strength γ [40], and the Raman-type atomic transition is triggered by two laser pulses with coupling strengths G' and κ [5], respectively. The co-existence of the atom-atom transition $|1\rangle \to |2\rangle$ and the atom-molecule transition $|1\rangle \to |3\rangle$ thus permits a competition between these two processes by starting from the same atomic sample. The focus of our present study is to see the role of initial populations imbalance on the closed-channel atom-molecule mutual coherence. In addition, for very short times or a dilute sample [13], we ignore the particles collisions to focus on the effects of coherent inter-mode couplings.

The interacting Hamiltonian of our model at the simplest level of single-mode approach (SMA) [41–43] can be written as ($\hbar = 1$)

$$\mathcal{H}_{int} = -\delta\hat{e}^\dagger\hat{e} + G'(\hat{e}^\dagger\hat{a}_1 + \hat{a}_1^\dagger\hat{e}) + \kappa(\hat{a}_2^\dagger\hat{e} + \hat{e}^\dagger\hat{a}_2) + \gamma(\hat{g}^\dagger\hat{a}_1\hat{b} + \hat{b}^\dagger\hat{a}_1^\dagger\hat{g}), \tag{1}$$

where δ is the intermediate state detuning in the atomic Raman transition, G' or κ is the optical pumping or dumping strength of the transition $|1\rangle \to |e\rangle$ or $|e\rangle \to |2\rangle$, and γ is the magnetic coupling strength of the transition $|1\rangle \to |3\rangle$. The operators $\hat{a}_i$ $(i = 1, 2)$, $\hat{e}$, $\hat{b}$ and $\hat{g}$ are the annihilation operators for the atoms A in the states $|i\rangle$ and $|e\rangle$, the atoms B in the state $|1\rangle$ and the formed heteronuclear dimers AB in the state $|3\rangle$, respectively. Obviously, a conserved quantity exists for the system: $\sum\hat{a}_i^\dagger\hat{a}_i + \hat{b}^\dagger\hat{b} + \hat{e}^\dagger\hat{e} + 2\hat{g}^\dagger\hat{g} = N_0$, where N_0 is the initial total atomic number.

The standard adiabatic approach by assuming δ as the largest evolution parameter of the system ($\dot{\hat{e}}/\delta \approx 0$) leads to the elimination of the intermediate atomic state $|e\rangle$:

$$\hat{e} \approx (G'\hat{a}_1 + \kappa\hat{a}_2)/\delta.$$

Then we can obtain an effective four-mode Hamiltonian with a linear atomic coupling term $G(\hat{a}_2^\dagger\hat{a}_1 + \hat{a}_1^\dagger\hat{a}_2)$, with $G = \kappa G'/\delta$. In the similar way, our model also can include a linear quasi-bond-bond coupling by introducing another dumping light [22, 23]. We note that the well-known coherent two-color PA Hamiltonian [44–48] can be viewed as just a specific case ($G = 0$) of our model here.

Let us first assume that the initial numbers of atoms A and B are small and their “depletion” effects (i.e., a large ratio of the initial atoms is converted into the closed-channel products) should be taken into account. The quantum dynamics and statistics of this nonlinear atom-molecule system are determined by the following Heisenberg equations of motion

$$\begin{aligned} &\frac{d\hat{a}_1}{dt} = iG\hat{a}_2 + i\gamma\hat{b}^\dagger\hat{g}, \qquad \frac{d\hat{a}_2}{dt} = iG\hat{a}_1, \\ &\frac{d\hat{b}}{dt} = i\gamma\hat{a}_1^\dagger\hat{g}, \qquad \frac{d\hat{g}}{dt} = i\gamma\hat{a}_1\hat{b}. \end{aligned} \tag{2}$$

We note that the loss of atoms from a Bose-condensed state generally occurs in very short time scales (up to two hundreds of μs) [48–51] and the upper limit of the heteronuclear molecules created in the present laboratory is only ∼ ms. This allows us to treat this nonlinear system by calculating the analytical solutions up to second order of the evolution time t [52]

$$\begin{aligned}
\hat{a}_1(t) &= \hat{a}_1 + itG\hat{a}_2 + it\gamma\hat{b}^\dagger\hat{g} + \frac{t^2}{2}\gamma\hat{a}_1\hat{g}^\dagger\hat{g} - \frac{t^2}{2}\gamma\hat{a}_1\hat{b}^\dagger\hat{b},\\
\hat{a}_2(t) &= \hat{a}_2 + itG\hat{a}_1 - \frac{t^2}{2}G^2\hat{a}_2 - \frac{t^2}{2}G\gamma\hat{b}^\dagger\hat{g},\\
\hat{b}(t) &= \hat{b} + it\gamma\hat{a}_1^\dagger\hat{g} + \frac{t^2}{2}G\gamma\hat{a}_2^\dagger\hat{g} + \frac{t^2}{2}\gamma^2\hat{b}\hat{g}^\dagger\hat{g} - \frac{t^2}{2}\gamma\hat{a}^\dagger\hat{a}\hat{b},\\
\hat{g}(t) &= \hat{g} + it\gamma\hat{a}_1\hat{b} - \frac{t^2}{2}G\gamma\hat{a}_2\hat{b} - \frac{t^2}{2}\gamma^2\hat{b}^\dagger\hat{b}\hat{g} - \frac{t^2}{2}\gamma^2\hat{a}_1\hat{a}_1^\dagger\hat{g},
\end{aligned} \tag{3}$$

from which one can see that $\sum \hat{a}_i^\dagger\hat{a}_i + \hat{b}^\dagger\hat{b} + 2\hat{g}^\dagger\hat{g} = N_0$, as it should be. With these general solutions, it is steadily to calculate the second-order quadrature squeezing of the closed-channel atomic and molecular fields. Our purpose is to verify the quantum transfer process between the initial squeezed atoms and the output molecules and even the possibility of non-locally entangled atom-heteronuclear molecule pairs [12, 53–55].

To this end, we first assume an initial coherent atomic ensemble, i.e., $|in\rangle = |\alpha\rangle_{a_1} \otimes |\beta\rangle_b \otimes |0\rangle_{g,a_2}$, where $|0\rangle_{g,a_2}$ denotes the initial vacuum state of the product matter-wave fields, $|\alpha\rangle$ and $|\beta\rangle$ are the Glauber coherent states:

$$\hat{a}_1|\alpha\rangle = |\alpha|e^{i\varphi_1}|\alpha\rangle, \qquad \hat{b}|\beta\rangle = |\beta|e^{i\varphi_2}|\beta\rangle.$$

By introducing the well-known squeezed coefficients [56]

$$S_i(t) = \frac{\langle(\Delta\hat{X}_i)^2\rangle - \frac{1}{2}|\langle[\hat{X}_1, \hat{X}_2]\rangle|}{\frac{1}{2}|\langle[\hat{X}_1, \hat{X}_2]\rangle|} \quad (i = 1, 2) \tag{4}$$

with two quadrature operators $\hat{X}_1 = \frac{1}{2}(\hat{a} + \hat{a}^\dagger)$, $\hat{X}_2 = \frac{1}{2i}(\hat{a} - \hat{a}^\dagger)$, we can readily calculate the squeezed coefficients for the output atoms A: $S_{1a,2a}(t) = 0$, which means that there is *no squeezing* for the atoms A in both of the two components S_{1a} and S_{2a}, even by taking into account of the depletions of the trapped atoms A and B. For the output dimers AB, we can get the same results: $S_{1g,2g}(t) = 0$, which indicates the *squeezing-free* effect in the two components S_{1g} and S_{2g} of the molecular field. This is exactly the place that quantum state transfer technique can be applied through initially preparing a squeezed atomic condensate [57]. We can see in the following that different squeezed effects can appear in the output matter-wave fields by starting from the squeezed atoms A or B.

Let us firstly start from a squeezed condensate of atoms A (through the efficient method of, e.g., Orzel et al. [58]), i.e., $|\alpha\rangle_s = \hat{S}(\xi_1)|\alpha\rangle$, where the squeezed operator $\hat{S}(\xi_1) = \exp(\xi_1^*\hat{a}_1^2 - \xi_1(\hat{a}_1^\dagger)^2)$, with $\xi_1 = \frac{1}{2}r_1e^{i\theta_1}$ $(r_1 > 0)$. For this case, we can calculate

$$\begin{aligned}
S_{1a,2a}(t) &= 2t^2G^2\sinh r_1(\sinh r_1 \pm \cos\theta_1\cosh r_1),\\
S_{1g,2g}(t) &= 2t^2\gamma^2\sinh r_1(\sinh r_1 \pm \cos\theta_1\cosh r_1),
\end{aligned} \tag{5}$$

where r_1 and θ_1 are the squeezed amplitude and squeezed angle, respectively. These results show that the squeezed behaviors remain the *same* for the atoms A and the heteronuclear molecules AB. In particular, the exhibition of the squeezed behaviors can be well controlled by the *squeezed angle* θ_1: if one choose $\theta_1 = 2n\pi$ $(n = 0, 1, 2, \ldots)$, then $S_{1a} = t^2G^2(e^{-2r_1} - 1) > 0$ and $S_{2a} = t^2G^2(e^{2r_1} - 1) < 0$, i.e., the component S_{2a} of the output atoms A is squeezed; but if one choose $\theta_1 = (2n+1)\pi$, then $S_{1a} < 0$, $S_{2a} > 0$, i.e., the quantum squeezing now transfers to the component S_{1a}; but for $\theta_1 = (n + \frac{1}{2})\pi$, one can obtain $S_{1a} = S_{2a} > 0$, i.e., there is always a *squeezing-free* effect for the output atoms A. For the output molecules AB, we have confirmed that similar results can be obtained also by steering the squeezed angle θ_1.

However, if one chooses to start from an initially squeezed condensate of atoms B, i.e., $|\beta\rangle_s = \hat{S}'(\xi_2)|\beta\rangle$, where the squeezed operator $\hat{S}'(\xi_2) = \exp(\xi_2^* \hat{b}^2 - \xi_2(\hat{b}^\dagger)^2)$ with $\xi_2 = \frac{1}{2} r_2 e^{i\theta_2}$ ($r_2 > 0$), the final results turn out to be

$$\begin{aligned} &S_{1a,2a}(t) = 0, \\ &S_{1g,2g}(t) = 2t^2\gamma^2 \sinh r_2 (\sinh r_2 \pm \cos\theta_2 \cosh r_2). \end{aligned} \tag{6}$$

It is clear that the squeezing effects for the output atoms A are different in this case, i.e., there is *no squeezing* now, although the interesting θ_2-dependent squeezing can still happen for the created molecules AB.

In the same way one can also study the two-mode atom-molecule correlations or entanglement. To this end, one can introduce the following second-order cross-correlation function [59]

$$\mathcal{G}_{ag}^{(2)}(t) = \frac{\langle a_2^\dagger(t) a_2(t) g^\dagger(t) g(t)\rangle}{\langle a_2^\dagger(t) a_2(t)\rangle \langle g^\dagger(t) g(t)\rangle} \tag{7}$$

where $\mathcal{G}_{ag}^{(2)}(t) > 1$ (or < 1) means the two-mode correlated (or anti-correlated) states while $\mathcal{G}_{ag}^{(2)}(t) = 1$ indicates no mutual correlations. According to Ref. [59], the violation of the following Cauchy-Schwarz inequality (CSI) for the two bosonic modes

$$[\mathcal{G}_{ag}^{(2)}(t)]^2 \le \mathcal{G}_{a}^{(2)}(t)\mathcal{G}_{g}^{(2)}(t), \tag{8}$$

with $\mathcal{G}_a^{(2)}(t) = \langle (a_2^\dagger)^2(t) a_2^2(t)\rangle / N_a^2$, $\mathcal{G}_g^{(2)}(t) = \langle (g^\dagger)^2(t) g^2(t)\rangle / N_g^2$, and $N_a = \langle a_2^\dagger(t) a_2(t)\rangle$, $N_g = \langle g^\dagger(t) g(t)\rangle$, will indicate the nonlocal violation of the Bell inequality or the appearance of macroscopic quantum entanglement.

For our present scheme, we particularly concern the role of different populations imbalances $\Delta \equiv N_{a_1}(0) - N_b(0)$ of the initial atoms A and B. For this purpose, we firstly study the specific case of nearly zero populations imbalance ($\Delta \sim 0$) as follows.

Case 1. With depletions of atoms A and B ($\Delta \sim 0$)

Using the general solutions (3), we can obtain that for an initial coherent sample ($|\alpha| = |\beta|$),

$$\mathcal{G}_{ag}^{(2)} = 1 + \frac{(2G^2 + \gamma^2 \mathcal{N} e^{i\varphi})t^2 - 2}{4|\alpha|^2}, \tag{9}$$

where we denote $\mathcal{N} = 2|\alpha|^2 - 1$ and $\varphi = \varphi_1 + \varphi_2$ for simplification. As a concrete example we assume $\varphi = 0$ and then for $t < 2\tau_0^{-1}$, $\tau_0 \equiv \sqrt{2G^2 + \gamma^2 \mathcal{N}}$, we have $\mathcal{G}_{ag}^{(2)} < 1$ or an anti-correlated atom-molecule state for the initial short-time evolution stage. In other words, the mutual correlations between the closed-channel atoms A and molecules AB are established and then changed dynamically in our present case. In addition, there is the violation of classical CSI or the appearance of nonlocal atom-molecule entanglement (for $|\alpha| > 1$), i.e.,

$$[\mathcal{G}_{ag}^{(2)}(t)]^2 - \mathcal{G}_{a}^{(2)}(t)\mathcal{G}_{g}^{(2)}(t) = \frac{1}{4}\left[4|\alpha|^2 - 3 + t^2\tau_0^2(2|\alpha|^2 - 1)\right] > 0. \tag{10}$$

Actually the non-classical features of the closed-channel atoms A and molecules AB can be further verified by calculating their Q parameters [59]

$$Q_{a,g}(\tau) = \frac{\langle \Delta \hat{N}_{a,g}^2(\tau)\rangle}{\langle \hat{N}_{a,g}(\tau)\rangle} - 1. \tag{11}$$

It is straightforward to calculate the related fluctuations $\langle \Delta \hat{N}_{a,g}^2\rangle = \langle \hat{N}_{a,g}^2\rangle - \langle \hat{N}_{a,g}\rangle^2$ and then we get

$$Q_a(\tau) = -t^2 G^2 < 0, \qquad Q_g(\tau) = -\frac{1}{2} t^2 \gamma^2 \mathcal{N} < 0, \tag{12}$$

which indicates that both of output atoms A and heteronuclear molecules AB can exhibit the non-classical sub-Poisson distributions [59].

Now we turn to the other two cases of large populations imbalance, i.e., $\Delta \ll 0$ or $\Delta \gg 0$. The former case corresponds to an initially large sample of atoms B and thus for very short times the slowly change of the population of atoms B can be ignored. One may refer this case as no depletion of atoms B.

Case 2. No depletion of atoms B (the slow change of atoms B in the trap) ($\Delta \ll 0$)

In this specific case we can ignore the slow change of atoms B in the trap, which means that the operators $\hat{b}_1^\dagger, \hat{b}_1$ can be replaced by a c-number $\sqrt{N_b}$ (Bogoliubov approximation). Then we can reach an effective linearized three-mode Hamiltonian, i.e.,

$$\mathcal{H}_{int} = G(\hat{a}_2^\dagger \hat{a}_1 + \hat{a}_1^\dagger \hat{a}_2) + \lambda(\hat{g}^\dagger \hat{a}_1 + \hat{a}_1^\dagger \hat{g}), \tag{13}$$

with $\lambda = \gamma\sqrt{N_b}$, which is actually a familiar linear three-well model with a factorized structure for the wave function [57]. This means that the initial coherent sample can not lead to any non-classical effect in the final products. With this point in mind, we turn to consider an initially squeezed condensate of atoms A and then it is easy to verify that we can reach (5) again, i.e., there are the same squeezed effects for the output atoms A and molecules AB.

In addition, with an initial squeezed vacuum of the atoms A (note that for this state $N_{a_1}(0) \neq 0$ [59]), one can obtain the two-mode correlation function

$$\mathcal{G}_{ag}^{(2)}(t) = 1 + \frac{(G^2+\lambda^2)t^2 + 8\cosh^2 r_1 - 4}{4\sinh^2 r_1} > 0, \tag{14}$$

$$[\mathcal{G}_{ag}^{(2)}(t)]^2 - \mathcal{G}_a^{(2)}(t)\mathcal{G}_g^{(2)}(t) \sim t^2(G^2+\lambda^2) + 12\sinh^2 r_1 + 4 > 0 \tag{15}$$

which means that in this quantum-transfer case the created two matter-wave fields (atoms A and dimers AB) are correlated. This is in contrast to the anti-correlated result as in the case 1 or (9). Besides, a perfect quantum transfer of the non-classicality of the entrance-channel atomic squeezing to the closed-channel atom-molecule entanglement can be realized in the present case.

Finally we have for the Q parameters

$$\begin{aligned} Q_a^s(\tau) &= (2\cosh^2 r_1 - 1)t^2\lambda^2 > 0, \\ Q_g^s(\tau) &= (2\cosh^2 r_1 - 1)t^2 G^2 > 0, \end{aligned} \tag{16}$$

which means that both of the atoms A and the molecules AB exhibit the super-Poisson distributions, which is again in contrast to (12) of the case 1.

Case 3: No depletion of atoms A (the slow change of atoms A in the trap) ($\Delta \gg 0$)

In this situation the number of atoms A are so large that we can ignore their slow change in the trap and therefore the operators $\hat{a}_1^\dagger$, $\hat{a}_1$ can be replaced by a c-number $\sqrt{N_a}$. The resulting effective Hamiltonian can be rewritten as

$$\mathcal{H}_{int} = \lambda_1(\hat{a}_2^\dagger + \hat{a}_2) + \lambda_2(\hat{g}^\dagger \hat{b} + \hat{b}^\dagger \hat{g}) \tag{17}$$

where $\lambda_1 = G\sqrt{N_{a_1}}$ and $\lambda_2 = \gamma\sqrt{N_{a_1}}$. For an interesting comparison, we start from an initially squeezed sample of atoms B and then obtain the squeezed coefficients for the final-state atoms A and molecules AB, which are found to be exactly in the form of (6), i.e., the θ_2-dependent squeezing effect only appears in the molecular field.

For the atom-molecule correlation function, however, we can get the following results

$$\mathcal{G}_{ag}^{(2)}(t) = 1, \tag{18}$$

$$[\mathcal{G}_{ag}^{(2)}(t)]^2 - \mathcal{G}_a^{(2)}(t)\mathcal{G}_g^{(2)}(t) = -1 - \cosh^2 r_2 \le 0, \tag{19}$$

which means that the atom-molecule correlation *never* can be established in this case. In fact this result can be directly observed in the form of the Hamiltonian (17). Also one can see that there exists no violation of the classical CSI even in the presence of a squeezed atomic sample.

We note that the fully quantum dynamics of our model also can be studied by, e.g., developing a set of stochastic equations for the matter-wave fields based on the positive-P representation technique in quantum optics [60, 61]. We actually have done this and found that the long-time behaviors of the particles populations are essentially classical and the mean-field results coincide with the quantized results in the short-time limits. The Gaussian-noise-term-induced molecular damping occurs only near a total atom-molecule conversion, a feature which was observed firstly in the context of atom-dimer conversion by Hope and Olsen [62]. We also note that, as the noiseless limit of the fully quantum Itô stochastic equations in the framework of positive-P representation [60, 61], the mean-field results about the well-known Rabi oscillations coincide with the quantized results in the short-time limits.

Finally we give a brief discussion about the possible influence of the decoherence effects on the proposed creation of atom-molecule entanglement. We note that the main source of decoherence includes the inevitable interactions between atoms and the electromagnetic vacuum, the spontaneous emissions, the fluctuation of the magnetic field and the losses of FR molecules. As Dannenberg and Mackie pointed out in their recent work [63], the rogue dissociative decoherence also can play an important role in any practical creation of atom-molecule superposition (the formation of superpositions is possible only when the decohering time scale is longer than particular interaction time scale [63]). In their analysis, the best chance of creating an atom-molecule superposition in a decohering system is for an atomic density $\sim 10^{12}$ cm^{-3} and temperature ~ 0.1 nK, which turns out to be challenging for current ultracold experiment. The similar obstacle also exist in the present scheme of atom-molecule entanglement, especially for the case 1 with obvious depletion or "consumption" of the initial particles (atoms A and B). In this case, the quantum entanglement is created due to the inherent nonlinearity of the atom-molecule coupling and thus the above upper bound for decoherence effects also exist here and remains a hard task for creating the stable atom-molecule entanglement. (The case 2, however, is reminiscent of the model describing the atom-light or atom-atom entanglement by applying an input squeezed light [54, 57, 58].)

For this reason, the practical nonideal issues (especially the existence of dissociation losses) will make the creation of atom-molecule entanglement an iffy problem in a practical system [20, 21, 63]. These complex issues will be studied in our future works. We emphasis that the main purpose of the present work is about the important role of different initial states and populations imbalance on the atom-heteronuclear dimer correlation, which exists at least in principle for an ideal or dilute atomic sample.

Summing up, we have studied the effects of initial state preparation and population imbalance in creating the quantum entanglement between the ultracold atoms and the heteronuclear molecules. We show that it is possible, at least in principle, to realize various interesting non-classical effects such as the super-/sub-Poisson distribution, squeezing/no-squeezing or correlated/anti-correlated effects in the created atom-molecule fields, indicating richer physics in creating the heteronuclear molecules. We plan to study the atom-molecule entanglement by starting from a boson-fermion mixture with large populations imbalance [43, 64–66] in our future work. We will also quantify the created atom-dimer entanglement and probe their possible applications in current quantum information science.

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